

# Optimal Energy Purchase in different Markets for Electric Automobile Battery Charging

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## 1 Introduction

We study the control of battery charging demanded by a large number of plug-in hybrid or all electric vehicles (PHEV) within an Electric Power System Control Area (EPSCA) for the energy service company (ESCO). Focusing on the load side participation in the day-ahead market and real-time market, a model is developed, coded and simulated to minimize the cost and risk of purchasing the energy to meet the battery charging requirements.

## 2 NYISO & PJM Market Framework

An operative market framework similar to that used in PJM (Pennsylvania-Jersey-Maryland market) and NY-ISO(New York Independent System Operator) will be assumed, i.e. we will model a somewhat simplified and stylized so as to retain the essential nature but not necessarily all details. The NYISO(and likewise PJM) includes two-settlement system consisting of the day-ahead market and the real-time balancing market. In day-ahead market, Enough generating units are committed and scheduled a day before operating to meet the forecasted load. In the real-time market, any real-time deviations from the day-ahead schedules are cleared. Based on [1][2][3], we describe the ISO below to provide an adequate perspective to the market model that we will use.

### 2.1 Day Ahead Scheduling

A Day Ahead Scheduling (DAS) where price-quantity pair offers are made by all market participants(on the generation and demand sides) and nodal hourly clearing prices and quantity schedules are determined a day in advance. Both load serving entities and generators bid into the market until 5:00 of the day before the operating day. The day-ahead market solution is made available by 11:00 of the day before the operation. Generators are paid nodal prices while loads are charged zonal prices in NYISO. There are 11 zones in NYISO and loads in a zone pay the weighted average of nodal prices at nodes that supply that zone. Bids include multiple block quantity price offers and bids. For generators start and shut down , minimum run time, down time, minimum, and maximum generation constraints etc are provided. All generators must submit bids into the Day-Ahead market or report their unavailable status. Virtual bids are allowed for virtual trades that are cleared in the real time market. Scheduling of generating capacity necessary for provision of regulation and operating reserves is performed simultaneously with scheduling of energy provision both in the

day-ahead and real-time markets. Unit commitment is also decided, particularly for units with longer than 30 min start time.

### 2.2 Real Time Scheduling

Balancing Market or Real Time Scheduling (RTS) Market: At  $h - (1 + 15\text{min})$  or  $(h - 2) : 45$  bids are locked and RTS market closes. Bids include multiple block quantity price offers and bids. For generators their ramp rates are also declared. Price sensitive loads that have demonstrated that they can respond to dispatch instructions, are dispatched the same way as generators are. The RTS market consists of two rolling-horizon market clearing components:

#### 2.2.1 Real Time Commitment

Real Time Commitment (RTC) runs every 15 minutes(0:00, 0:15, 0:30, 0:45, 1:00, 1:15, . . . h:00, h:15, . . .) with a 2 hour 15 minute look ahead horizon beyond the end of the next 15 min interval, that is, each run of RTC evaluates the next ten points in time separated by fifteen-minute intervals using the same set of bids(please refer to figure 1). The purpose of the RTC is to adjust the day-ahead schedules of the external interchanges (exports and imports into the zone) and to make adjustments to the day-ahead commitment of generation resources with start-up time within 10 to 30 minutes and with minimum up-time within the horizon of the RTC. Specifically, a RTC run initiated at  $t - \Delta t$  so that it posts results at time  $t$ , which is posting time. RTC uses bids locked at time  $(t-1) : 45$  and posts commitments and advisory clearing prices for the next ten 15min intervals, namely  $t$  till  $t + 0 : 15, t + 0 : 15$  till  $t + 0.30, t + 2 : 15$  till  $t + 2 : 30$ . The posted commitments for intervals  $t + 0 : 15$  till  $t + 0 : 30$  and  $t + 0 : 30$  till  $t + 0.45$  are binding. However, all clearing prices are advisory! The energy and ancillary service prices will be ex post, based on the actual performance of the units on dispatch during the prior interval(rather than ex ante, based on the expected performance of these units).

#### 2.2.2 Real Time Dispatch

Real Time Dispatch (RTD) runs every 5 minutes, optimizing over a horizon of 60 minutes. For the RTD time line, please refer to figure 2. RTD makes no unit-commitment decisions and it only performs an economic dispatch of units that have been previously committed by the day-ahead market or by the RTC. The schedule for interval  $t$  till  $t + 5$  min interval is binding, and during this interval it calculates congestion reflecting and

generation dynamic constraints reflecting nodal energy prices. These nodal energy prices are used in the financial settlement of deviations between actual operating levels and those specified in the DAS. Energy prices estimated for the remaining 10,15,15,15 min intervals in the hour are advisory.

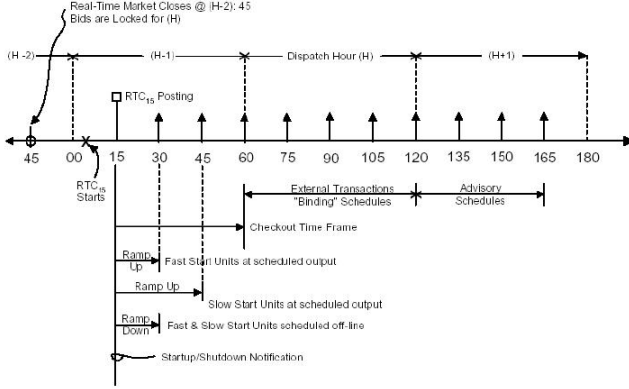


Figure.1 Real Time Commitment, Source:[3]

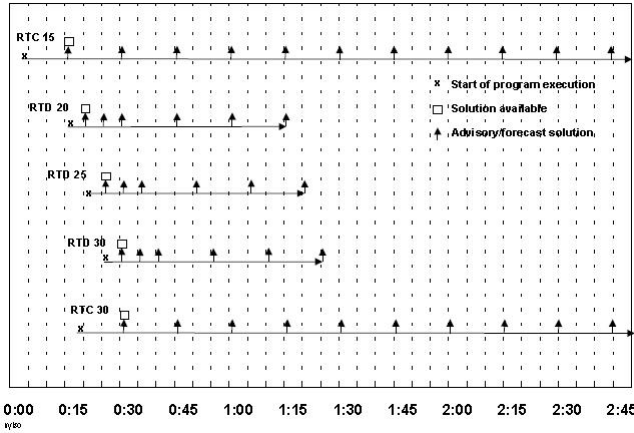


Figure.2 Real Time Dispatch, Source:[4]

### 2.3 Ancillary Services

Regulation Service plus (in NYISO) three types of Operating Reserve Markets (10-min spinning reserve, 10-min reserve, and 30-min reserve), i.e. four ancillary service markets are cleared simultaneously with the energy market. Regulation is needed to counter the minute to minute rises and falls in electricity consumption, and operating reserves are needed to offset possible forced outages of generating or transmission facilities. Since there are two energy markets, the DAS and the RTD market, all four ancillary services are cleared simultaneously with energy in each of the two markets.

## 3 Energy Service Company Decision Framework

We assume that electric vehicles' owners sign contracts with an Energy Service Company (ESCO) to have their batteries charged by a deadline specified and communicated to the ESCo every time each electric vehicle is plugged into an electricity outlet through a smart interface installed on it.

In the day-ahead market, there are two decisions that ESCo must make:

1. How to allocate purchase in day-ahead market and real-time market to minimize the total purchasing cost while keeping the risk low.
2. How to submit demand bids in the day-ahead market.

As time progresses, with more information available, in the real-time market, there are also two decisions the ESCo must make:

1. If real demand deviates from the expected one, Escos needs to buy additional energy or sell excess energy back in the real-time market. The decision is to buy or sell that amount of energy in the current operating hour or the following ones, whose energy prices may be different from the current one. Using the smart interfaces to control the charging rate, We can do this as long as these vehicles can be fully charged by the deadline.
2. How to submit demand bids in the real-time market.

## 4 Electricity Market Model Employed

Electricity markets which we are studying consist of a day-ahead market and a real-time energy balancing market. In both markets, generation-side and demanding-side bid into the markets simultaneously. In market structures in which demand-side market is not implemented, the load serving entities(LSE) should bid a fixed demand. In other markets, LSEs can bid a non-increasing bidding curve into the markets. Bidding curves can be piecewise linear or stepwise linear.

It is important for us to know the market clearing process. Based on[5][6][7] The popular modal for the ISO is represented as follows:

Let  $m$  power suppliers bid linear supply curve denoted by  $P = a_i + b_i S_i, i = 1, 2, \dots, m$ , where  $P$  is the market clearing price and  $S$  is the power generation. Let  $n$  large consumers bid linear demand curve denoted by  $P = c_j - d_j L_j, i = 1, 2, \dots, n$ , where  $L$  is consumption. Let the aggregate load from small users be  $Q = Q_0 - KP$ , where  $K$  is the price elasticity of small customer.

The dispatching target of ISO is to maximize social welfare and balance the supply and demand. Therefore,

$$\sum_{i=1}^m S_i = Q_0 + \sum_{j=1}^n L_j \quad (1)$$

Where

$$S_i = \frac{P - a_i}{b_i} \quad \text{for supply curve} \quad (2)$$

$$L_j = \frac{c_j - P}{d_j} \quad \text{for demand curve} \quad (3)$$

Power generation and consumption limit constraints

$$S_{\min,i} \leq S \leq S_{\max,i} \quad (4)$$

$$L_{\min,j} \leq L \leq L_{\max,j} \quad (5)$$

Market clearing price is determined from (1),(2) and (3)

$$P = \frac{Q_0 + \sum_{i=1}^m \frac{a_i}{b_i} + \sum_{j=1}^n \frac{c_j}{d_j}}{K + \sum_{i=1}^m \frac{1}{b_i} + \sum_{j=1}^n \frac{1}{d_j}} \quad (6)$$

Using  $P$  from (6) in (2) and (3), if  $S_i$  or  $L_j$  is below its limit, remove that generator or consumer from the system and calculate  $P$  again. Similarly, if  $S_i$  or  $L_j$  is greater than its limit, set  $S_i$  or  $L_j$  to its upper limit and calculate  $P$  again by ignoring that generator or consumer since it is no longer a marginal unit. Continue this process until power produced and consumed by each firm is within the limit, MCP is finally obtained.

## 5 Purchase Allocation and Bidding Curve Generation in the Day-ahead Market

As we mentioned before, in the day-ahead market, ESCo should make a plan to optimize its purchase allocation in the day-ahead market and the real-time market in order to minimize its cost.

Like [8][6], we can write down a simple cost function of ESCo in one hour

$$C = x \cdot D \cdot P_1 + (1 - x) \cdot D \cdot P_2 \quad (7)$$

$C$ —total cost of purchasing power  
 $P_1$ —day-head market clearing price  
 $P_2$ —real-time market clearing price  
 $D$ —total energy purchased (expected)  
 $x$ —ratio of the energy purchased in the day-ahead market to the total energy

In the above cost function,  $P_1$  and  $P_2$  can be considered as random variables (because the market clearing price is ex post). Moreover, due to sequential market clearing, the price of the second market is usually related to the price of the first market[9]. The problem is a stochastic optimization problem in which  $x$  should be determined so that the total cost will be minimized.

Notice that  $x$  can be larger than 1. In this case, ESCo sells back the excess energy in the real-time market. If the real-time clearing price is higher than the day-ahead market clearing price, ESCo will make profit by ordering more energy than its real demand ( $x > 1$ ). But since  $P_1$  and  $P_2$  are unknown until ISO posts the market clearing price which may take place several days after, ESCo and other LSEs may face certain risk of losing money. In NYISO, to ensure market liquidity the two-settlement system allows free arbitrage between the day-ahead and the real-time markets. For that purpose the day-ahead market accepts virtual bids, that is, offers to supply or bids to purchase energy in the day-ahead market not associated with actual generating or load resources. However, in some markets LSEs are expected to submit bids in accordance with the load that they expected in the day-ahead market. The reason is that the ISO does not like demand-side speculation in real-time market. If the LSEs deviate from this rule, they will be penalized. Also, if a large portion of demand is to be

purchased in real-time market, LSEs might not be fully dispatched. Therefore, we add a term to the cost function, and it becomes

$$C = x \cdot D \cdot P_1 + (1 - x) \cdot D \cdot P_2 + \eta \cdot (1 - x)^2 \quad (8)$$

where  $\eta$  is a positive coefficient.

$P_1$  and  $P_2$  are random variables. We use normal distributions to describe the behavior of these random variables. Since  $P_2$  is conditional on  $P_1$ , we write

$$P_1 \sim N(\mu_1, \sigma_1^2) \quad (9)$$

$$P_2 \sim N(\mu_2, \sigma_2^2) \quad (10)$$

$$\mu_2 = E[P_2|P_1] \quad (11)$$

$$\sigma_2^2 = E[(P_2 - \mu_2)^2|P_1] \quad (12)$$

$$\sigma_{12}^2 = E[(P_1 - \mu_1)(P_2 - \mu_2)|P_1] \quad (13)$$

The expectation of the total purchase cost is

$$\begin{aligned} \bar{C} &= E_{P_1, P_2} [C(P_1, P_2)] \\ &= E_{P_1, P_2} [xD P_1 + (1 - x)D P_2 + \eta(1 - x)^2] \\ &= xD\mu_1 + (1 - x)D\mu_2 + \eta(1 - x)^2 \end{aligned} \quad (14)$$

From the well developed investment theory, it is known that the variance of the potential profit could be used to evaluate the risks of an investment. Here, Risk is reflected by the variance of the cost

$$\begin{aligned} &E_{P_1, P_2} [(C(P_1, P_2) - \bar{C})^2] \\ &= E_{P_1, P_2} \{[(xD P_1 + (1 - x)D P_2 + \eta(1 - x)^2 \\ &\quad - xD\mu_1 - (1 - x)D\mu_2 - \eta(1 - x)^2)]^2\} \\ &= E_{P_1, P_2} \{[(xD(P_1 - \mu_1) + (1 - x)D(P_2 - \mu_2))]^2\} \\ &= x^2 D^2 E_{P_1, P_2} [(P_1 - \mu_1)^2] + (1 - x)^2 D^2 E_{P_1, P_2} [(P_2 - \mu_2)^2] \\ &\quad + 2x(1 - x)D^2 E_{P_1, P_2} [(P_1 - \mu_1)(P_2 - \mu_2)] \\ &= D^2 [x^2 \sigma_1^2 + (1 - x)^2 \sigma_2^2 + 2x(1 - x)\sigma_{12}^2] \end{aligned} \quad (15)$$

Thus, our problem is formulated as follows:

min  $J$ ,

$$\text{With } J = E_{P_1, P_2} [C(P_1, P_2)] + q E_{P_1, P_2} [(C(P_1, P_2) - \bar{C})^2] \quad (16)$$

where  $q$  is a risk weighting factor.

In the above formulation, not only the prices but also the price variations are considered in purchase allocation. The selection of  $q$  is dependent on ESCo's goal and tolerance on the risk. This is similar to balanced stock portfolio allocation with consideration on the expected return and risk.  $q$  can be determined by experience.

Then

$$\begin{aligned} J &= xD\mu_1 + (1 - x)D\mu_2 + \eta(1 - x)^2 \\ &\quad + qD^2 [x^2 \sigma_1^2 + (1 - x)^2 \sigma_2^2 + 2x(1 - x)\sigma_{12}^2] \\ \frac{1}{D} \frac{\partial J}{\partial x} &= \mu_1 - \mu_2 + 2 \frac{\eta}{D} (x - 1) + 2qDx(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}^2) \\ &\quad + 2qD(\sigma_{12}^2 - \sigma_2^2) \end{aligned}$$

Let  $\frac{\partial J}{\partial x} = 0$ . Then

$$\mu_1 - \mu_2 - 2\frac{\eta}{D} + 2qD(\sigma_{12}^2 - \sigma_2^2) + [2\eta + 2qD(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}^2)]x = 0$$

because  $\eta > 0, q \geq 0, D > 0, \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}^2 \geq 0$ ,  $J$  reaches its minimum when

$$x = \frac{2\frac{\eta}{D} + \mu_2 - \mu_1 + 2qD(\sigma_2^2 - \sigma_{12}^2)}{2\frac{\eta}{D} + 2qD(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}^2)} \quad (17)$$

Like [9][10], let us consider a special case in which the expected price of the real-time market is linear correlated with the price of the day-ahead market, that is,

$$\begin{aligned} \mu_2 &= E[P_2|P_1] = \alpha P_1 + \beta \\ \sigma_{12}^2 &= E[(P_1 - \mu_2)(P_2 - \mu_2)] \\ &= E[P_1 P_2] - \mu_1 \mu_2 \\ &= E_{P_1} [E_{P_2} [P_1 P_2 | P_1]] - \mu_1 \mu_2 \\ &= E_{P_1} [P_1 (\alpha P_1 + \beta)] - \mu_1 \mu_2 \\ &= \alpha(\sigma_1^2 + \mu_1^2) + \beta \mu_1 - \mu_1(\alpha \mu_1 + \beta) \\ &= \alpha \sigma_1^2 \end{aligned} \quad (18)$$

Then

$$x = \frac{2\frac{\eta}{D} + (\alpha - 1)\mu_1 + \beta - 2qD(\sigma_2^2 - \alpha\sigma_1^2)}{2\frac{\eta}{D} + 2qD(\sigma_1^2 + \sigma_2^2 - 2\alpha\sigma_1^2)} \quad (19)$$

Next step is to generate a bidding curve for the day-ahead market. We use (19) to generate such a bidding curve. Here, we take some data to build a bidding curve. Given

$$\begin{aligned} D &= 100\text{MWH}, \mu_1 = 60\$/\text{MWH}, \sigma_1 = 100, \sigma_2 = 200 \\ \alpha &= 0.9, \beta = 8, \eta = 30, q = 6 \times 10^{-5} \end{aligned}$$

We can get

$$x = 0.6275$$

which means that we should purchase 62.75% of our total demand, that is, 62.75 MWH in the day-ahead market.

But in order to obtain a curve, for example, a line, we need at least two points. Therefore, we take another set of data

$$\begin{aligned} D &= 100\text{MWH}, \mu_1 = 62\$/\text{MWH}, \sigma_1 = 110, \sigma_2 = 220 \\ \alpha &= 0.9, \beta = 8, \eta = 30, q = 6 \times 10^{-5} \end{aligned}$$

Then we obtain

$$x = 0.4341$$

Using these two points, we can generate a bidding line. Figure.3 shows the bidding line generated by the two points.

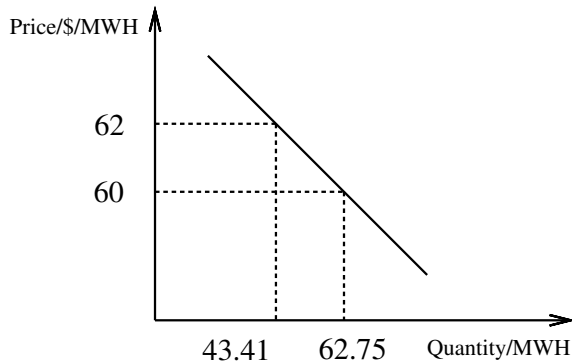


Figure.3 Bidding line

So far, we have done the strategy for the day-ahead market. Next, we will deal with the strategy for real-time market.

## 6 Strategy and Bidding Curve Generation in the Real-Time Market

In the real-time market, for hour  $t$  and  $t + 1$ , the market clearing prices of the day-ahead market  $P_1(t)$  and  $P_1(t + 1)$  are already known. Suppose that at the beginning of hour  $t$ , ESCo receives messages from all the vehicles that reports their status. Then ESCo can make an estimation of the amount of energy that it should buy (if the demand is more than the energy scheduled) or sell (if the demand is less than the energy scheduled) in the real-time market. Let that amount of energy be  $\Delta D$ , then

$\Delta D \geq 0$ , ESCo buys energy in the real-time market

$\Delta D \leq 0$ , ESCo sells energy in the real-time market

Because we are charging the electric vehicles who have smart interfaces that can control the charging rate, we can actually charge more in hour  $t$  and charge less in hour  $t + 1$ , or charge less in hour  $t$  and charge more in hour  $t + 1$ . Basically, the market clearing prices are usually different for that two hours. Thus, it provides us an opportunity to optimize the allocation of  $\Delta D(t)$  in the two-hour period ( $t$  and  $t + 1$ ).

For simplicity, we do that optimization for two-hour period. At the beginning of hour  $t$ , ESCo observes  $\Delta D(t)$  and allocates one part of  $\Delta D(t)$  in the real-time market of the hour  $t$  and the other part of  $\Delta D(t)$  in the real-time market of hour  $t + 1$ . Then, at the beginning of hour  $t + 1$ , ESCo observes  $\Delta D(t + 1)$  and do the same optimization for the hour  $t + 1$  and  $t + 2$ . So on and so forth.

Next, we will find out the optimal allocation. The information available to us at the beginning of hour  $t$  is

1. The day-ahead market clearing prices  $P_1(t)$  and  $P_1(t+1)$
2. As we mentioned in the section of Real Time Scheduling (RTS), ISO will post the advisory prices for the next 2 hours and 30 minutes. Thus, we have  $P_{AD}(t)$  and  $P_{AD}(t + 1)$
3. noise  $v(t)$  ( $v(t + 1)$ ) that affects the real-time market clearing price, with zero mean and  $\sigma_v^2$  variance.

We can express  $P_2$  as

$$P_2 = k_1 P_1 + k_2 P_{AD} + v \quad (20)$$

With

$$\mu_2 = k_1 P_1 + k_2 P_{AD} \quad (21)$$

$$\sigma_2^2 = \sigma_v^2 \quad (22)$$

Where  $k_1$  and  $k_2$  are coefficients and  $k_1 + k_2 = 1$ .

We can write down the cost function

$$C = y \cdot \Delta D P_2(t) + (1 - y) \cdot \Delta D P_2(t + 1) \quad (23)$$

Where  $y$  is the proportion of energy to be ordered or sold in the real-time day market of the hour  $t$ .

Then, we will

$$\begin{aligned} & \min J, \\ & \text{With } J = \mathop{E}_{P_2(t), P_2(t+1)} [C(P_2(t), P_2(t+1))] \\ & + q \mathop{E}_{P_2(t), P_2(t+1)} [(C(P_2(t), P_2(t+1)) - \bar{C})^2] \end{aligned} \quad (24)$$

where  $q$  is a risk weighting factor.

Finally, we obtain

$$\begin{aligned} y = & \frac{\sigma_v^2(t+1)}{\sigma_v^2(t+1) + \sigma_v^2(t)} \\ & + \frac{k_1[P_1(t+1) - P_1(t)] + k_2[P_{AD}(t+1) - P_{AD}(t)]}{2qD(\sigma_v^2(t+1) + \sigma_v^2(t))} \end{aligned} \quad (25)$$

We use (25) to generate a bidding curve for the real-time market. Again, we take two sets of data to build a bidding curve.

Given Set 1

$$\Delta D = 30\text{MWH}, P_1(t) = 60\$/\text{MWH}, P_1(t+1) = 61\$/\text{MWH}$$

$$P_{AD}(t) = 61\$/\text{MW}, P_{AD}(t+1) = 62\$/\text{MW}$$

$$\sigma_v^2(t) = 10, \sigma_v^2(t+1) = 15, k_1 = 0.2, k_2 = 0.8, q = 6 \times 10^{-3}$$

We can get

$$y = 0.7111, yD = 0.7111 \times 30 = 21.333$$

$$\mu_2(t) = k_1 P_1(t) + k_2 P_{AD}(t) = 60.8$$

Given Set 2

$$\Delta D = 30\text{MW}, P_1(t) = 62\$/\text{MW}, P_1(t+1) = 63\$/\text{MW}$$

$$P_{AD}(t) = 62\$/\text{MW}, P_{AD}(t+1) = 64\$/\text{MW}$$

$$\sigma_v^2(t) = 20, \sigma_v^2(t+1) = 25, k_1 = 0.2, k_2 = 0.8, q = 6 \times 10^{-3}$$

We can get

$$y = 0.6667, yD = 0.6667 \times 30 = 20.001$$

$$\mu_2(t) = k_1 P_1(t) + k_2 P_{AD}(t) = 62$$

Using these two points, we can generate a bidding line. Figure.4 shows the bidding line generated by the two points.

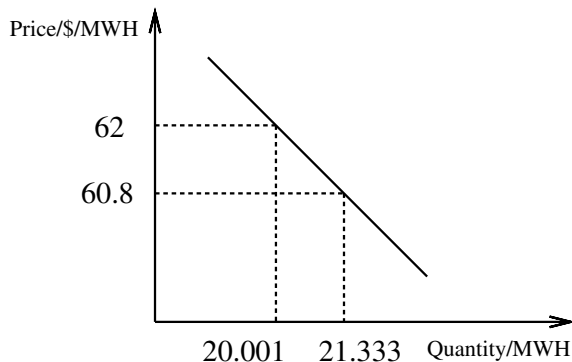


Figure.4 Bidding line for the real-time market

## 7 Strategy for ESCo

In fact, there lies a problem for the above method. For instance, if  $\Delta D$  is 30MWH, after optimization, we will charge all the vehicles that are plugged in 20MWH in the hour  $t$  and charge them 10MWH in the hour  $t+1$ . The problem is that people may use the vehicle after the hour  $t$ , but the vehicle has not been charged as much as possible. To solve this problem, we have 2 ways:

1. Once the vehicle is plugged in, the owner indicates when he or she will use it through the smart interface. ESCo can make all the vehicles into several groups according to their urgency and charge different groups with different power.
2. Do the optimization in a smaller time scale. We can divide the time line by 15-min interval instead of one hour interval, since the RTS also offers advisory price every 15 minutes.

## 8 Simulation

Here, we downloaded the data from the website of NYISO as following tables.

Table.1

Day-Ahead Market	Zone	Price(\$/MWH)
04/16/2009 15:00	Central	34.91
04/17/2009 15:00	Central	33.97
04/18/2009 15:00	Central	30.67
04/19/2009 15:00	Central	27.85
04/20/2009 15:00	Central	33.54
04/21/2009 15:00	Central	33.4
04/22/2009 15:00	Central	31.57
04/23/2009 15:00	Central	34.99

Table.2

Real-Time Market	Zone	Price(\$/MWH)
04/16/2009 15:00	Central	16.06
04/17/2009 15:00	Central	27.83
04/18/2009 15:00	Central	15.58
04/19/2009 15:00	Central	27.1
04/20/2009 15:00	Central	33.32
04/21/2009 15:00	Central	32.38
04/22/2009 15:00	Central	42.33
04/23/2009 15:00	Central	36.52

The data above contains the day-ahead and real-time prices at 3:00pm from April 16th to April 23rd in the central zone. By simple calculation, we can get the means and variances (subscript 1 indicates day-ahead market and subscript 2 indicates real-time market)

$$\mu_1 = 32.61\$/\text{MWH}, \sigma_1^2 = 5.97, \sigma_2^2 = 88.11$$

To find out the linear correlation between the day-ahead price and the real-time price, we use linear regression. Then we can write down

$$\mu_2 = 0.34P_1 + 17.61$$

Next, based on this information, we will find out the optimal purchase allocation in the day-ahead market and real-time

market. Given

$$D = 100\text{MWH}, \eta = 750, q = 6 \times 10^{-5}$$

We can get

$$x = 0.6372$$

In practice, we need to know how much money this allocation can save. Here, we use Monte-Carlo simulation to do this task. The number of Monte-Carlo simulation is specified to be 5000. We compare the traditional method and the proposed method. The traditional method is that buy the expected amount of energy in the day-ahead market ( $D$ ) and buy or sell  $D' - D$  in the real-time market, where  $D'$  is the demand observed in the real time. The proposed method is that buy  $x \cdot D$  amount of energy in the day-ahead market and buy or sell  $D' - xD$  in the real-time market. Let  $D' \sim N(100, 30)$ . Here, we only do optimization of the allocation between the day-ahead and real-time market. The results are showed below. (cost is average cost)

Table.3

$q$	$x$	Traditinal method(\$)	Proposed Method(\$)	Cost reduced(%)
$2 \times 10^{-5}$	0.7119	3258.1	3153.8	3.20
$6 \times 10^{-5}$	0.6372	3265.2	3127.4	4.22
$8 \times 10^{-5}$	0.6024	3266.2	3115.0	4.63
$1 \times 10^{-4}$	0.5690	3261.4	3095.0	5.10

From the table above, as the risk weighting factor  $q$  increases, the purchase allocation in the day-ahead market is reduced and the average cost is reduced too. However, how to set the value of the risk weighting factor is dependent on ESCo's goal and tolerance on the risk. From the historical data, we know that the real-time market is more volatile than the day-ahead market. Allocating more purchase in the real-time market is more risky in short-term.

## 9 Conclusion

In this paper, the purchase allocation in both the day-ahead and real-time markets for the energy service company(ESCO) with risk consideration is discussed. Simple and reasonable solutions for optimal purchase allocation and bidding curve generation are obtained. The possible battery charging strategy for the ESCo is also discussed. Numerical simulation is performed based on the data from NYISO. The costs using Traditional method and the proposed method are Compared. In the future, the electric vehicles will definitely become a mainstream because of their efficiency and low pollution. This method of optimal battery charging will has its potential in the near future.

## Appendix A. PJM and NYISO

Table.4 Comparison of markets between NYISO and PJM

	New York ISO	PJM
Number of markets	Day-ahead, hour-ahead, and realtime	Day-ahead and realtime
Market products	Energy plus four ancillary services	Energy plus regulation and spinning reserve
Locational reserve pricing	Yes	No
Energy bids	3-part	3-part
Day-ahead unit commitment	Voluntary, central	Voluntary, central
Congestionmanagement pricing	Nodal for generation, zonal for load	Nodal for generation and load
Losses	Marginal	Average
Integration of markets	Yes	Partial
Installed capability requirements	Yes	Yes

## Appendix B. Introduction to the Monte-Carlo Simulation

The Monte-Carlo method simulated the functions of a real system randomly generates stochastic variables, and then opens out the running laws of the studied system. The essence of Monte-Carlo method is provide numerical approximate solutions to mathematical, physics and engineering problems by performing stochastic simulations and statistical sampling experiments on a computer. The method is directly applicable to problems with inherent probabilistic structures, and requires that the physical or mathematical system be described by probability density functions.

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